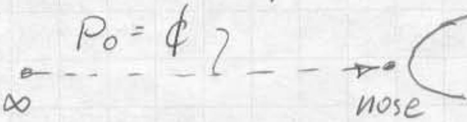
UE Fluids F14 Solution S'08

a) $a_\infty = \sqrt{\gamma p_\infty/\rho_\infty}$, $\boxed{M_\infty \equiv V_\infty / a_\infty = V_\infty \sqrt{\rho_\infty/(\gamma p_\infty)}}$

b) Assuming no bow shock (subsonic flight), $p_{o_{nose}} = p_{o_\infty} \equiv p_o$



$$p_{o_{nose}} = p_{o_\infty} = p_\infty\left[1 + \frac{\gamma-1}{2}M_\infty^2\right]^{\frac{\gamma}{\gamma-1}} \quad (p_o \text{ definition})$$

Using M_∞ result from a) ...

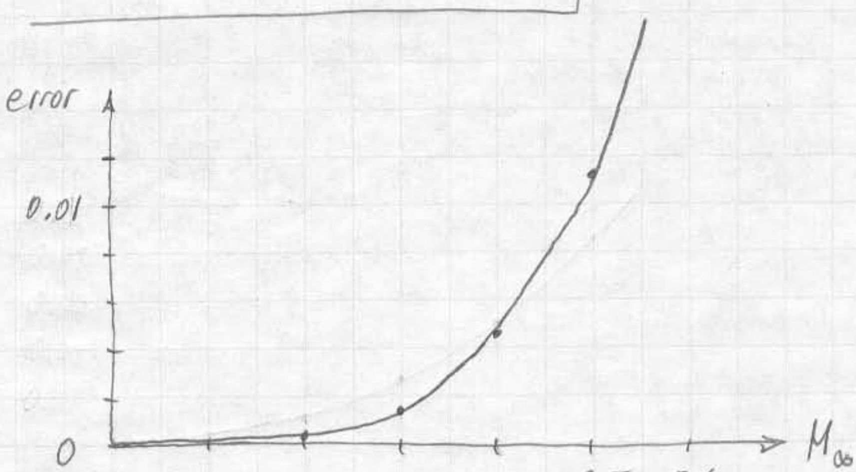
i) $$\boxed{p_{o_{nose}} = p_\infty\left[1 + \frac{\gamma-1}{2}\frac{\rho_\infty V_\infty^2}{\gamma p_\infty}\right]^{\frac{\gamma}{\gamma-1}}}$$

ii) Using Bernoulli (incorrectly) : $p_o = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2$

or $$\boxed{p_o = p_\infty\left[1 + \frac{\gamma}{2}\frac{\rho_\infty V_\infty^2}{\gamma p_\infty}\right] = p_\infty\left[1 + \frac{\gamma}{2}M_\infty^2\right]}$$

$$\boxed{\text{error} = \left(\frac{p_o}{p_\infty}\right)_{i)} - \left(\frac{p_o}{p_\infty}\right)_{ii)} = \left[1 + \frac{\gamma-1}{2}M_\infty^2\right]^{\frac{\gamma}{\gamma-1}} - \left[1 + \frac{\gamma}{2}M_\infty^2\right]}$$

M_∞	error
0	0
0.1	0.0000175
0.2	0.00028
0.3	0.00143
0.4	0.00455
0.5	0.0112
0.6	0.0235



$M_\infty < 0.27$ gives $\approx 0.1\%$ accuracy , seems pretty safe

$M < 0.47$ gives $\approx 1\%$ accuracy , starts to get significant